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The Pirate Game - A Logic Puzzle

Five pirates arrive back on board ship with 100 gold coins and decide to share them out using a democratic system. Each of the pirates has a different rank, and the most superior pirate will therefore take the first turn at proposing the way in which the gold is shared out. The pirates will then have a vote, which, if it is a draw, will be won by the pirate making the proposition. If the proposal is accepted, the gold is shared out, but if not, the pirate making the proposal is thrown overboard and the next in the chain of command is allowed to propose a way to share out the gold. These are, in fact, the rules for a *mathematical game*[1] known as the 'pirate game', and the result is quite different from what the average person would expect.

ENTRY DATA

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Referenced Entries:

Prisoner's Dilemma
John Forbes Nash - Mathematician
An Introduction to Game Theory
Gold
Legends of Pirates and Piracy

Related BBC Pages:

BBC Radio 4 - Puzzles

The Problem

We will label the pirates *A*, *B*, *C*, *D* and *E*, with *A* being the highest-ranking and *E* being the ship's dogsbody. Though they are supposedly your run-of-the-mill scourge of the sea and are naturally very self-centred, the pirates all have a good grasp of the situation and can anticipate what the others will do. A pirate offered nothing in a proposal will vote against it, as they have nothing to lose but wouldn't mind seeing another pirate thrown to the sharks. One would expect the resulting shares to look something like this:

- *A* - 20
- *B* - 20
- *C* - 20
- *D* - 20
- *E* - 20

However, this seemingly obvious solution is ruined by the presence of the chain of command and the seemingly innocuous voting system. In actual fact, the majority of the crew end up agreeing to *A* taking almost all the gold. This seems illogical, but the following will show how it is completely true. The solution is best explained by looking at what each pirate would propose, starting with the dogsbody, *E*.

D's and *E*'s Proposals

E never actually gets to make a proposal, although he would award every single gold piece to himself if he had the chance. The problem is that *D*'s proposal comes first, and *D* will always win this ballot as he holds the casting vote. Therefore, if pirates *A*, *B* and *C* were to be cast overboard, *D* would be able to propose the following and win:

- *D* - 100

- *E* - 0

C's Proposal

Being a reasonably intelligent pirate, *C* knows that if he gets to make a proposal through both *A* and *B* being thrown overboard, he would only have to offer *E* better than nothing to win the vote. Offering *D* better than nothing wouldn't work, as *D* knows that if *C* goes overboard, *D* would get the whole stash. *C* could therefore propose the following and win:

- *C* - 99
- *D* - 0
- *E* - 1

B's Proposal

Also being reasonably intelligent, *B* has got out a small piece of paper and worked out all the above. To win the vote, all *B* has to do is to offer *D* more than he would get if *C*'s proposal was adopted. Although *D* would be able to take the lot if *A*, *B* and *C* were to go overboard, *D* realises that in reality the game would never get that far, as *C* and *E* would gang together to make *C*'s proposal win, as stated above. *B* can therefore offer *D* one more coin than he would get if *B* were thrown overboard, thus allowing *B* to win with the casting vote. The winning proposal would therefore be:

- *B* - 99
- *C* - 0
- *D* - 1
- *E* - 0

A's Proposal

At this point, *A* looks over at *B* scribbling on his small piece of paper and, with a smug grin, makes the following proposal:

- *A* - 98
- *B* - 0
- *C* - 1
- *D* - 0
- *E* - 1

Knowing that, if given the chance, *D* will help *B* win *B*'s proposal and leave the others with nothing, *C* and *E* happily vote for *A*'s proposal, thus allowing *A* to walk off with the fortune after a democratic vote. Brilliant, isn't it?

A Nash Equilibrium

When played with five players who all know what they're doing, the game has but one possible outcome, in which the most superior of the five takes the majority share. This is due to the fact that this solution is a Nash equilibrium. Put simply, this is where no player can benefit from changing their individual strategy while the others keep theirs the same, and so none of the players will do so, thus making the current result inevitable. In the pirate game the equilibrium is said to be unique as there is only one possible result which leads to the equilibrium. However, reaching this solution requires a certain set of circumstances:

- All the players are rational.
- All the players assume that all their adversaries are rational as well.
- The players are intelligent enough to realise the solution.
- The players all understand the pay out they are competing for and will attempt to maximise their own gains.
- The players all carry out their strategies without making mistakes.

Therefore, the game will not necessarily produce the same results in real life, and is at best restricted to a world containing rational beings, which we are generally not. Making these assumptions about the pirates is necessary for the Nash equilibrium to be reached, as player *A* requires so-called 'perfect information' in order to deduce exactly what the others will be thinking. Perfect information basically describes a situation where each player knows all the relevant facts about the others and can therefore make use of the best strategy possible. *A*'s strategy is known as the 'best response', as it guarantees him the biggest payout possible under the conditions given. The responses of the other pirates are also best responses, although unfortunately they do not lead to particularly good results. However, each player makes use of the best strategy available, and that is why we end up with a Nash equilibrium.

More General Nash Equilibria

The availability of perfect information is only necessary for the successful creation of a unique Nash equilibrium such as that in the pirate game - in other games, the conditions for a Nash equilibrium can be much simpler:

- When one of the players changes their strategy without any of their opponents altering theirs:
  - The player who changed their strategy is worse off.
  - The players who did not change their strategies are not offered a better potential strategy through this change, and will therefore keep their original strategies.

Provided this is the case, the player will realise that they stand to lose out and will change back to their best response strategy. An example of this can be seen in the Prisoner's Dilemma, in which both players may either co-operate or betray one another. While co-operating brings a good reward, if one player co-operates and the other betrays, the second wins a slightly higher prize while the first gets nothing. However, if both players betray, they end up with only a small payout each. Since the players do not know exactly what the other player will do, perfect information is not available. However, a Nash equilibrium exists when both players choose to betray the other. In this situation, if one player were to change stance while the other player stuck with betraying, the player making the change would lose out, while the other player would stand to gain from sticking to the strategy of betraying. This means that the game can end up in a deadlock, with neither player daring to cooperate lest they let the other player simply capitalise through continued betrayal. Both players will therefore continue to accept a minimal payoff, thus proving that rational game strategies aren't always the most sensible.

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[1] That is to say one which is used in game theory to

model aspects of real life.

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